New trends for the Kondo effect in nanostructures

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Abstract: The Kondo effect in confined nanostructures (quantum dots) provides a testbed for a variety of physical behaviors involving strong electronic correlations. Here some extensions of the Kondo effect beyond the standard single-impurity Anderson model are reviewed. Apart from their fundamental interest, these issues may also open new roads for low-temperature spintronics.

Keywords: quantum dots; electronic confinement; Kondo effect; decoherence; electronic interferometry; Josephson junctions; quantum criticality; spintronics; spin filtering.

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1 Introduction

The Kondo effect is a milestone of magnetism in metals. The appearance of an upturn followed by a low-temperature saturation of the resistivity in metals containing diluted magnetic impurities was explained by Kondo in 1960 as an enhanced scattering due to a screening of the local impurity spins by the conduction electrons [1]. The Fermi liquid nature of the ground state [2] and the various theoretical treatments, including Wilson’s numerical renormalization group [3], make the Kondo problem one of the best understood phenomena in Condensed Matter, as well as a paradigm for correlated

electron physics [4].
In the late 90's, the prediction by Ng and Lee [5], Glazman and Raikh [6] of the Kondo effect in a small quantum dot coupled to Fermi liquid leads boosted a revival of the subject. Here, confinement, interactions and electrostatic gating conspire to achieve a finely tunable artificial Anderson impurity. Instead of a maximum of resistivity in the limit of zero temperature as in bulk metals, in the present case an applied bias forces the electrons to flow through the Kondo resonance, thus producing a maximum (unitary) conductance $2e^2/h$ at zero temperature [7]. In fact, although the microscopics of the Kondo effect in a quantum dot is basically the same as in the bulk, quantum dots offer a new system to investigate the Kondo effect, most exclusively through electrical transport measurements. Following the initial theoretical prediction, it has been observed in 2D GaAs/AlGaAs heterostructure quantum dots [8,9], silicon MOSFET's [10-11], as well as in carbon nanotubes [12] and contacted single molecules [13-14]. The smaller the quantum dot, the larger its energy scales and also possibly the Kondo temperature, which can reach several dozens of Kelvin in single molecules.
Quantum dots not only allow to tune all the parameters of the Anderson model, but they offer a large panel of extensions of the Kondo effect, such as the introduction of new environmental degrees of freedom or more complex impurity states, as well as out-of equilibrium situations, which are striking examples of the versatility of the Kondo physics in quantum dots. We briefly review here some theoretical advances achieved in the Rhône-Alpes laboratories. Without quoting the full bibliography for each topic, we sketch the main results of these challenging issues, some of them being under experimental investigation, others being the object of theoretical predictions.
Moreover, the Kondo effect in nanostructures allows to bridge a very fundamental effect with nanoelectronics devices. Indeed, one can take advantage of the quantum coherence properties of the Kondo effect by conceiving set-ups that could fulfill some basic functions of spintronics, as spin filters or possibly quantum gates or single spin manipulations.
This short review is organized as follows: Part II summarizes several extensions of the single-impurity Kondo problem: i) breaking the spin symmetry with a magnetic field or polarized leads; ii) connecting the dot to a resistive environment; iii) exploring the Kondo cloud through finite-size effects in the leads; iv) embedding the dot in a ring to perform quantum interferometry; vi) connecting the dot to superconducting leads. In part III, more exotic Kondo effects, due to the inclusion of other degrees of freedom, are reviewed: i) Kondo-driven quantum phase transitions; ii) SU(4) and orbital Kondo behavior with possible application as a spin filter.

2. An Anderson single-impurity in new environments

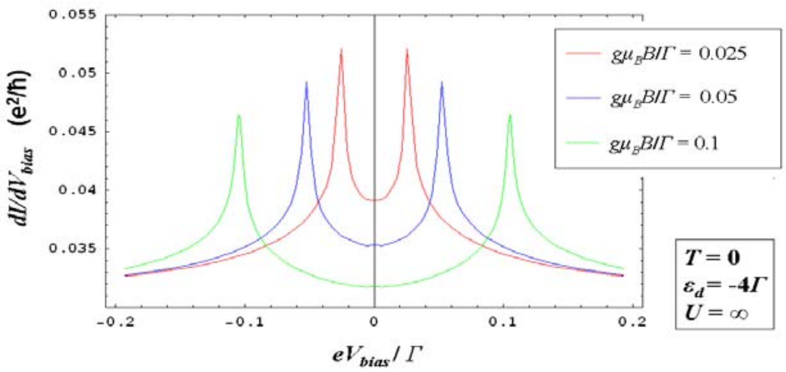
2.1 Kondo effect with a magnetic field or ferromagnetic leads

The Kondo effect requires spin degeneracy of the localized electron level. When an external magnetic field breaks this spin degeneracy, the Kondo peak shifts away from the chemical potential by the Zeeman energy $g\mu_B B$, but in opposite direction for each spin: this is known as the field-induced splitting of the Kondo resonance. This effect has been observed experimentally in the non-equilibrium transport through a quantum dot in the presence of a magnetic field [8,9]. At zero magnetic field, the differential conductance as a function of bias voltage exhibits a peak at zero bias (zero-bias anomaly) when $T<T_K$. By applying a parallel magnetic field, the zero-bias peak is split into two side peaks, separated approximately by twice the Zeeman energy. This is in contrast with the conventional resonant tunnelling, where the peak separation is about the Zeeman energy. This doubling of the distance between the split peaks in dI/dV in the presence of a magnetic field, is viewed as a hallmark of the many-body physics of the Kondo effect.

The effect of a magnetic field in the non-equilibrium regime of a Kondo quantum dot has been addressed in a number of papers using a large variety of techniques. In a pioneering work, Meir, Wingreen and Lee (MWL) [15] addressed this question by using the equation-of-motion technique combined with the non-crossing approximation (NCA) calculations for the determination of the occupation numbers. Their calculations are performed at infinite interaction U and pushed up to the second order perturbation in the dot-lead coupling $V_{L,R}$ (left L and right R lead indices). In its initial form, the approach leads to logarithmic singularities in the localized electron spectral density. To provide a physical cut-off for the logarithmic divergence, MWL proposed to introduce a finite lifetime to the excited states based on the use of Fermi's golden rule at the 4^{th} order in $V_{L,R}$. Then their predictions lead to the onset of two side peaks distant of twice the Zeeman energy. Other calculations using numerical renormalization group (NRG) [16], Bethe ansatz [17], or perturbation theory [18] approaches in the non-equilibrium regime predict a value of the splitting in dI/dV lying between $g\mu_B B$ and $2g\mu_B B$. However, these theoretical studies are contradicted by recent experimental results by Kogan *et al* [19] who, using a new method of inelastic spin flip cotunneling, unexpectedly measured a Kondo splitting exceeding $2g\mu_B B$ by about 10%.

In a recent work, we have revisited the problem using the equation-of-motion decoupling technique but pushed it up to the 4^{th} order in $V_{L,R}$ [20]. It is shown that our method naturally introduces a finite lifetime of the excited states whenever a magnetic field or a bias voltage is applied. Although it confirms the intuitive argument of MWL to introduce a physical cut-off to the logarithmic divergence, based on the use of Fermi's golden rule, our self-consistent approach shows that the lifetimes of both spins combined are necessary to account for the cut-off, instead of only that of the opposite spin. We report the differential conductance as a function of the bias voltage in Fig. 1 at different values of the magnetic field

Figure 1 Differential conductance dI/dV versus bias voltage Vbias at different values of the magnetic field B.



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It reveals a peak separation in the differential conductance due to the magnetic field. Work is currently in progress to make a quantitative comparison between the peak splitting and the Zeeman energy with a view to explaining recent contradictory experimental results. Numerical simulations based on improved NRG calculations at finite-energy [21] are also currently underway.

As shown in several works [22,23,24], coupling a small quantum dot to spin-polarized (ferromagnetic) reservoirs leads similarly to a splitting of the Kondo resonance for strong enough polarizations. It has been shown that this effect can be compensated by applying a Zeeman field, which means that both – spin polarization in the leads and Zeeman field acting on the impurity – affect the Kondo resonance in a similar way.

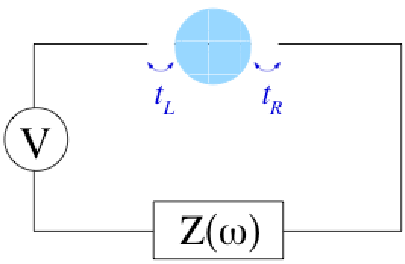
Since there are two reservoirs, their relative spin polarization is an additional control parameter. We have explored the regime of non-collinear polarizations [25] and shown by poor man's scaling arguments that the Kondo temperature is maximal when those are antiparallel, and minimal when they are parallel. In fact, in the former case, spin symmetry is restored if the coupling to the two leads are identical. NRG calculations show that splitting of the Kondo resonance occurs for strong enough polarizations as soon as they are not antiparallel. It can also be compensated by applying an auxiliary magnetic field acting on the impurity.

2.2 Kondo effect in a resistive environment

In the Kondo effect, the bare impurity level is out of resonance, and together with the large Coulomb repulsion the charge degree of freedom is thus nearly frozen to the addition or removal of one electron on the dot. Virtual charge fluctuations, denoted as cotunneling, provide a transport channel through the dot as well as a spin-exchange channel between the impurity spin and the conduction electron spins. The exchange is antiferromagnetic and leads to a divergence in perturbation theory, yielding a singlet ground state. In presence of two leads, the exchange interaction is made of two pieces: one involves each lead once at a time (J_{LL} and J_{RR}), the other (J_{LR}) involves both spin flip and electron transfer between L and R. The consequence of the Kondo spin screening is the formation of a charge resonance through which the double junction formed by the dot becomes fully transparent at T= 0.

On the other hand, any junction (unless perfectly transparent) or quantum dot placed in a resistive circuit is known to exhibit the Dynamical Coulomb Blockade (DCB) phenomenon: for an ohmic environment with a resistance approaching $R_K=h/e^2$, the junction transparency will be quenched, which manifests in a nonlinear current-voltage characteristics and vanishing conductance at low temperatures [26,27].

Figure 2 Schematics of the device studied in [28]: a small quantum dot carrying a magnetic moment, coupled to metallic reservoirs featuring an ohmic environment (taken from [28]).



Considering the same kind of environment in the Kondo regime, competing effects arise: on one hand, a perfectly transparent junction is known to be insensitive to an ohmic environment, so the T=0 fixed point should be a priori preserved. On the other hand, any kind of inelastic process, for instance due to finite temperature, affects the dot's transparency. In this case DCB should quench the linear conductance. This problem was solved in the limit where the dot charging energy is large enough so that the cotunneling processes through the dot level remain elastic, despite the ohmic environment. Yet, cotunneling events transferring a charge from L to R, as the Kondo process J_{LR}, are affected by the ohmic bath, just like in a single tunnel junction. As a result, this process tends to be quenched, while the other exchange processes J_{LL} and J_{RR} remain unchanged. As shown in [28], this has two generic effects. First, a weak coupling analysis shows that weak environmental fluctuations do not destroy the screening of the impurity spin, but simply lower the Kondo temperature. On the other hand, a mapping on a Luttinger liquid with a magnetic impurity shows that the low-energy processes leading to the unitary conductance are quenched, yielding a non-linear I(V) characteristics and DCB. The results depend on the parameter $r = R/R_K$. If $r < 1/2$, non-linear behavior characteristic of DCB appears. Only for the case of particle-hole symmetry the unitarity is preserved, but with anomalous deviations. If $r > 1/2$, the coupling J_{LR} vanishes in the low-energy limit and coherent electronic transfer is no longer possible between the leads. The special case of symmetric junctions $J_{LL} = J_{RR}$ provides a possible realization of the two-channel Kondo model, here due to DCB rather than true Coulomb blockade (see Section III.1).

As a remark, no direct proof of the "spin" manifestation of the Kondo effect – the formation of a singlet – exists up to now in nanostructures, in contrast to the evidence for the unitary conductance as a signature of the Kondo effect in transport properties. The above study shows that an ohmic environment allows to "decouple" these two effects, e.g. achieving the Kondo screening without linear conductance. Experiments testing this behavior require tunable highly resistive ohmic environment [29], and should probe anomalous scalings in the voltage bias and magnetic field, which are still not quantitatively understood theoretically.

Further investigations have included the extension to capacitively coupled quantum dots, characterized by entangled spin and orbital degrees of freedom [28]. As described in Section III.2, such devices provide the possibility of realizing efficient spin-filtering operations by static gate-voltage manipulations. In view of potential applications in the context of quantum information processing the question arises how a dissipative environment may affect the spin-orbital Kondo effect and, as a consequence, reduce the potential performance of logical spin operations. The resulting low-energy physics exhibits a rich phase diagram as a function of r and the geometric parameters of the system. Using a perturbative renormalization-group approach several crossover regions between different Kondo fixed points could be identified, in addition to a distinctive localized phase where spin-flip processes are fully suppressed for large values of r. The corresponding Kondo temperature allows to assess the robustness of the device against environmental effects. These are minimized at low bias voltage and for highly symmetric devices concerning the geometry.

2.3 Kondo effect in a finite-size environment: evidencing the screening cloud

The usual Kondo model applies to situations where the electronic reservoir (or the transport leads) connected to the small dot (the artificial Anderson impurity) is infinite, and described by a smooth density of states at the Fermi energy. If on the contrary the reservoir is finite, so that its density of states varies on the scale of the Kondo temperature T_K, new effects have been predicted [30,31], which directly reflect the existence of the Kondo screening cloud, an elusive object up to nowadays experiments. Since the size of the conduction electrons wavefunction screening the local "impurity" spin is of the order of $\xi_K = hv_F/ T_K \approx 1$ μm (in a 2DEG with $T_K \approx 1K$), modern lithographic techniques are available to build devices which are sensitive to this finite-size physics.

One possibility would be to probe the persistent currents in a ring closed by a small quantum dot. One expects that these decrease with a scaling law in ξ_K /L when the size L of the ring is smaller than ξ_K [30]. Such experiments are difficult, alternatively measuring the charging curve of a mesoscopic wire coupled to a small dot could be considered [31]. A third possibility (in principle easier) involves transport measurements through the system formed by two mesoscopic wire segments coupled through a small dot [32]. The wire segments exhibit density of states maxima, and the Kondo temperature of the system becomes modulated according to the position of the Fermi level of the wires with respect to these maxima. This modulation can be detected as a variation of the conductance at intermediate temperatures.

In such a set-up, the reservoirs play both the role of the probe electrons participating to transport, and of the screening electrons that bind to the local impurity spin. In a three-terminal geometry, those two effects can be separated. Transport occurs through two reservoirs L and R, weakly coupled to the small dot. The latter is instead strongly coupled to a finite-size reservoir N. In these conditions, the Kondo screening is essentially due to the mesoscopic reservoir N, while the transport leads L and R only act as a spectroscopic probe of the dot density of states for the combined "dot + N"system. The behaviour of the LR conductance displays original features, depending on whether the reservoir Fermi level is at resonance or off resonance with the density of states peaks [32,33]. This can be obtained by gating the mesoscopic reservoir N independently from the small dot. In particular, the conductance versus temperature displays a monotonous decrease in the off-resonant case, but is non-monotonous in the resonant case.

2.4 Transmission phase through a quantum dot and Aharonov-Bohm quantum interferometry

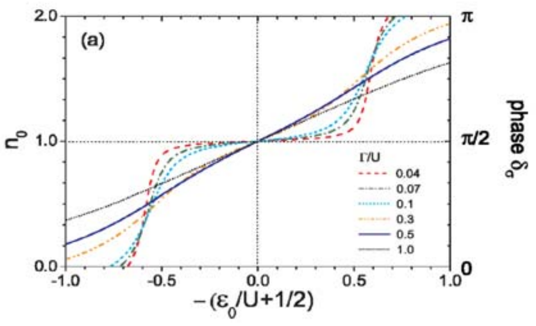
An important characteristic of the Kondo effect is the phase shift δ_{QD} an electron undergoes when it crosses a Kondo impurity. A direct measurement was not feasible in bulk systems but could be achieved in quantum dot realizations of the impurity, using Aharonov-Bohm (AB) interferometry. The determination of the phase is based on the interference between electrons traversing the two arms of an interferometer, one of which containing the quantum dot and the other one being the reference arm, with transmission amplitudes $t_{QD}=|t_{QD}|\exp[i\delta_{QD}]$ and t_{ref}, respectively, and a weak magnetic field threading the ring. The coherent current collected at the drain shows AB periodic oscillations as a function of the magnetic flux allowing the study of the dependence of the magnitude and the phase of the transmission amplitude on the various parameters of the system.

This has been studied in a remarkable series of experiments realized by the Heiblum group at the Weizmann Institute [34-38]. In relatively large quantum dots (typically with a number of electrons in the dot N larger than 14), the phase evolution is found to exhibit a 'universal' behavior [37] independent of the dot size, shape and occupancy.

Specifically, in the Coulomb blockade regime, the transmission phase increases from 0 to π throughout each conductance peak, returning sharply to its value 0 (complete phase lapse) in the conductance valley located between two consecutive peaks. In contrast, in small quantum dots (typically with N smaller than 10), the phase evolution shows a ‘mesoscopic’ behavior with the appearance or not of a phase lapse which strongly depends on the number of electrons in the dot. Whereas the first measurements [35,36] in small quantum dots dealt with the situation where more than a single level is involved, the most recent experiment [38] has finally succeeded in working with a single orbital state in the Kondo regime. In such a dot, the results show an increase of the phase shift from 0 to π/2 throughout the first transmission peak, followed by the presence of a plateau of value π/2 in the odd conductance valley between both transmission peaks, and then by another increase from π/2 to π throughout the second transmission peak. The understanding of the crossover from ‘mesoscopic’ to ‘universal’ behavior as the dot gets larger, constitutes a challenge for theorists.

It is believed that the single level Anderson model (SLAM) is appropriate to the study of the single orbital quantum dot, whereas the multi-level Anderson model (MLAM) is relevant to the study of larger quantum dots, considering decreasing values of δ/Γ as the quantum dot gets larger (where δ is the average level spacing). In the case of the SLAM, early theoretical work [39] for the bulk Kondo effect predicts a phase shift of value π/2 in the Kondo limit. Gerland *et al* [40], on the basis of numerical renormalization group and Bethe ansatz calculations, recovered this result in the context of quantum dots. We report in Fig. 3 the results obtained later on by Jerez *et al* [41] combining Bethe ansatz calculations and the use of Friedel sum rule $n_{0\sigma}=\delta_\sigma/\pi$, for the dependence of the occupation number $n_{0\sigma}$ on the normalized energy of the dot level at different values of the normalized dot-lead coupling. Considering a linear dependence of the dot level with the gate voltage, these predictions, contradicted by the first AB interferometry measurements [35,36], have been finally confirmed by the latest experiment [38]. With hindsight, the reason for the earlier discrepancy is probably that in the first experiments the condition of transport through only a single level was not realized, whereas in the latter one it was.

Figure 3 Theoretical results (taken from [41]) from the Bethe Ansatz calculations and the use of Friedel sum rule $n_{0\sigma}=\delta_\sigma/\pi$, for the dependence of the occupation number n_0 on the normalized energy of the dot level $[-(\varepsilon_0/U+1/2)]$ at different values of the normalized dot-lead coupling Γ/U. One notes that $n_0=1$ corresponding to $\delta_\sigma=\pi/2$ in the symmetric limit $\varepsilon_0=-U/2$, with the existence of a plateau in its vicinity at weak coupling. On the other hand, at larger coupling, δ_σ climbs almost linearly with the normalized energy from 0 to π.



As far as larger quantum dots are concerned, there has been an intensive theoretical effort to elucidate the origin of the “universal” behavior mentioned above starting from a MLAM. Various studies based on mean-field [42,43], numerical [44] or functional RG [45] approaches, lead to an increase of the phase from 0 to π throughout each conductance peak as expected for the transmission through a Breit-Wigner resonance. The consecutive complete phase lapse (sharp return of the phase to its value 0) in the conductance valley located between two consecutives peaks, is interpreted as resulting from zero transmission [46]. But then a naïve argument in the single-particle picture would lead to the conclusion that the corresponding phase lapse should occur only when the sign of the matrix elements coupling a given level to the left or right lead, is positive. As this is expected to occur randomly, a sign problem should occur. This sign problem, however, is found to be cured as soon as correlation effects are considered, providing an explanation for the generic features of the observed ‘universal’ behavior in large quantum dots [42-45].

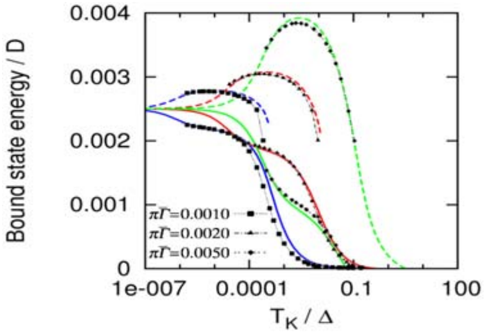
2.5 A dot connected to superconducting leads

A very interesting issue is raised when coupling the quantum dot to superconducting reservoirs [47,48]. This has been achieved in Grenoble with carbon nanotubes [49] and single fullerene molecules [50]. In the strong coupling conditions allowing the Kondo effect, the interplay between the latter and superconductivity is subtle and manifests itself in anomalous properties of the so-formed Josephson junction. A naive guess would be that the superconducting gap quenches the low-energy excitations and prevents the complete screening of the molecular spin. Yet, when T_K is much larger than the gap Δ, a Kondo resonance can still form, and still show a BCS feature at low energy. In this situation, the spin singlet formed with the reservoir electrons is compatible with singlet pairing and the junction shows standard properties: the minimum energy is obtained for a zero phase difference between the leads.

On the other hand, when T_K is much smaller than the gap Δ, the impurity spin screening does not occur and one is left with an essentially unscreened impurity spin. The situation is therefore similar to the case of a weakly coupled quantum dot, where Cooper pairs crossing the impurity spin take an additional phase π [51]. The current-phase characteristic inverts itself, leading to a so-called π-junction. The two states (“0” or “π”) of the junction have different spin symmetries (S= 0 or S = ½ respectively), and a first-order phase transition is obtained, either by varying the ratio T_K / Δ or by varying the phase difference for an intermediate ratio of T_K / Δ [52].

This problem has been attacked theoretically in the last fifteen years with a variety of methods, with the difficulty of spanning a wide range of physical regimes and parameters, in particular the position of the bare dot level with respect to the superconducting gap. Up to now, NRG has provided the most accurate results, but a complete and physical picture of superconducting quantum dots is still lacking, especially when several levels or dots are involved. One issue considered recently is the connection between atomic or molecular states and the Andreev Bound States (ABS) that live in the induced superconducting gap on the dot. A simple theory illustrated in Fig. 4, based on dressed ABS, was proposed to enlighten how ABS emerges from atomic levels, and gives quantitative predictions for future spectroscopic measurements with STM on carbon nanotubes connected to superconducting leads.

Figure 4 Evolution of the Andreev Bound State energies as a function of the ratio of the Kondo temperature over the superconducting gap, for several values of the hybridization, according to the calculation developped in [53]. Symbols are numerical data taken from [52].



Other current theoretical developments concern the generalization of Fermi Liquid Theory for quantum impurities in the presence of superconductivity, based on the adiabaticity of the Kondo screened ground state (the “0”-junction) to the non-interacting limit [53].

3. Exotic Kondo effects and quantum phase transitions

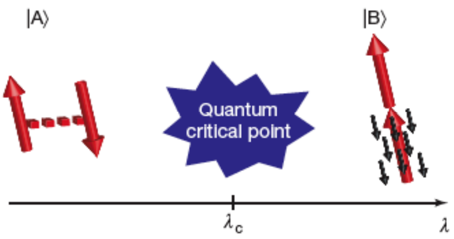
3.1 Quantum phase transitions

The possibility of designing and fine tuning the properties of magnetic impurities in quantum dots provides new handles to construct in the lab model Hamiltonians with interesting properties. One important focus is now on quantum-driven phase transitions between competing ground states, which are at the heart of many open issues in strongly correlated materials, and for which artificial nanostructures provide a simpler yet quite promising setting. Several examples have been achieved in recent experiments, following guidelines provided by the theory.

A first situation concerns transitions between zero-entropy phases in which the ground state is a spin singlet, which can be realized either by the competition between two electronic reservoirs, each trying to compensate a single spin ½ (the so-called two-channel Kondo problem [54]), or, in a double dot setup, by exploiting the possibility that two spins can be locked together in an inter-dot singlet or be Kondo screened by separate electrodes (the so-called two-impurity Kondo problem [55]). In both cases, the crucial limitation for the experiments to succeed lies in the transfer of charge between the two electronic reservoirs, which smoothen the expected quantum critical behavior. In the two-channel situation, it has been shown that charging effects can be used (in a similar spirit as discussed in section II-2) to preserve a true phase transition [56], as was demonstrated in recent experiments [57]. Concerning the two-impurity problem, a proposal to use up to four quantum dots in series was shown on theoretical grounds to suppress the similarly annoying tunneling processes [58], and this opens a new way for future experimental studies. Indeed, as expected in the presence of charge transfer processes, measurements on double quantum dots have up to now only show broad crossovers in the conductance [59], rather than singularities associated to a true phase transition.

In a second proposal, the so-called singlet-triplet transition [60], one of the accessible ground state involves a partially Kondo screened spin S=1 [54], which can be realized in even-charge multi-orbital quantum dots with large Hund's rule coupling, in the case where a single screening channel is at play (otherwise the spin is fully compensated and the transition turns again into a crossover).

Figure 5 Sketch of the singlet-triplet transition in quantum dots (taken from [61]).



The quantum critical point results then from a dissociation of the spin 1 local moment into two distinct spin ½ entities (e.g. by tuning the binding energy of the triplet relative to the singlet) that are separately screened. Such a scenario has received recent confirmation in an experiment done in Grenoble [61], and further theoretical work is under way along this line, in particular to understand the magnetic field response of underscreened Kondo impurities.

Finally, other types of proposals that were recently studied concern quantum phase transitions induced by a depletion of the low-energy excitations at the Fermi level, so that the Kondo effect competes with other possible unscreened ground states. This so-called pseudogap Kondo problem, initially proposed in [62], has been the focus of many recent analytical and numerical work, as very rich quantum critical behavior can be found [63]. This model is relevant for experiments involving magnetic impurities in zero gap semiconductors like graphene, or, in a similar spirit, in superconductors with nodal quasiparticles like high-Tc materials, but both offer limited tunability to explore this physics. Interestingly, artificial quantum dot setups can be devised as well to produce such vanishing density of states [64], which could motivate new experiments.

3.2 Coupled quantum dots and the entangled orbital-spin Kondo effect

Nanostructures allow to engineer novel set-ups extending the range of the Kondo effect. For instance, going beyond the spin degree of freedom only, one can use an orbital part of the electronic wavefunction, as pertaining to the same quantum dot (as in carbon nanotubes [65]), or to neighbouring quantum dots [66]. An example consists of two capacitively coupled dots coupled to two metallic reservoirs, such that four states (two spin values ↑, ↓, and two orbital states 1, 2, lead to the fourfold multiplet |1↑>, |2↑>, |1↓>, |2↓> at large intra- and inter-dot repulsion) can in principle be tuned to degeneracy. The fully degenerate situation leads to SU(4) (or SU(5)) Kondo effect.

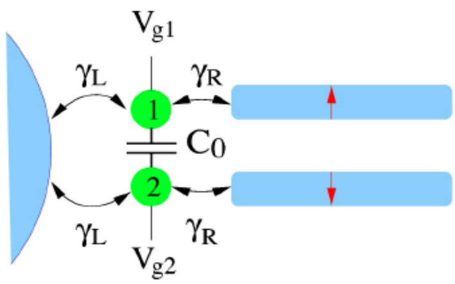
An interesting situation is obtained when the spin in the dots is polarized by a strong magnetic field, such as a pure orbital Kondo effect occurs, with, say the states 1↑, 2↑ being degenerate [67]. An experimental difficulty consists in achieving a strong enough capacitive coupling between nearly equivalent quantum dots. Kondo screening of the orbital state in the dots is obtained through two equivalent orbital states in the leads, and unitary conductance e^2/h is achieved, with full spin polarization at low energy. This spectacular effect can be used as a principle for an efficient spin filter [67]. Notice that the Kondo effect is not necessary for achieving strong spin filtering: weakly coupled dots have been used for this purpose, leading to efficient filtering but low transparency. Conversely, the Kondo regime allows to achieve both high polarization and high transparency, both properties being locked together in the quantum coherence of the Kondo resonance. A necessary condition is that the Kondo temperature needs to be smaller than the Zeeman splitting. Another requirement is that other orbital levels should lie at higher energy in order to avoid mixing to triplet states.

We have shown that when tuning the gates applied to each of the dots, and applying a strong Zeeman field, one can set at degeneracy the two states 1↑, 2↓. Here the different gate voltages applied on dots 1, 2 compensates for the Zeeman splitting. Under the same experimental constraints as above, one can then achieve *entanglement* of the spin and the orbital degrees of freedom. Thus, the low-temperature Kondo state achieves entanglement both in the sense of a singlet formed between the dots and the reservoirs as well as (a novel feature) spin-orbital entanglement. Notice that the Kondo resonance is split into two spin-polarized components living each one in a different dot (spin ↑ in dot 1 and spin ↓ in dot 2) [68].

One can take advantage of spin-orbital entanglement to achieve a novel type of *bidirectional* spin filter: connecting the two dots in parallel to the same source L but to two distinct drains R1, R2 (see Fig. 5), one can in such a three-terminal geometry achieve a Stern-Gerlach spin filter. A spin-up polarized current flows between L and R1, while a spin-down polarized current flows between L and R2. The efficiency and the transparency of the filter is again determined by the "quality" of the Kondo effect, reaching the unitary limit $G_{1\uparrow} = e^2/h$, $G_{1\downarrow} = 0$, $G_{2\downarrow} = e^2/h$, $G_{2\uparrow} = 0$ when $T \ll T_K$.

Notice that any asymmetry in the couplings between each of the dots and the leads introduces the equivalent of a spin splitting. This can be compensated by finely tuning the gate voltages and reaching the ideal Kondo effect, without changing the Zeeman field [68].

Figure 6 Schematics of a spin splitter based on a double dot, capacitively coupled, in a three-terminal geometry (taken from[68]). The gate voltages select spin ↑ (in dot 1) and spin ↓ (in dot 2) transitions respectively, and the orbital Kondo effect ensures the transparency and efficiency of the filter.



The quantum coherence of this set-up can be exploited in another viewpoint, that of quantum information. Entanglement of spin and orbital means that any phase shift applied on the orbital degree of freedom can be swapped onto the spin. This can be achieved in an Aharonov-Bohm geometry, where the two coupled dots in parallel are connected to a single source and drain. The Aharonov-Bohm phase applied by tuning a small magnetic field normal to the 2DEG – while the Zeeman field is parallel to it – yields a coherent *precession* of the spin across the device [69]. Such an experiment would be the elecronic equivalent of a famous Gedanken experiment, initially proposed by Bohm [70], then realized with polarized neutrons [71]. Such a spin precession, whose angle is controlled by the orbital magnetic field is an example of how an elementary quantum gate can be achieved with a static and tunable parameter configuration of the system.

4. Conclusion

The work reported in the present review contributes to extending the physics of the Kondo effect in several directions, well beyond the phenomenology known in the 80's in diluted magnetic alloys. It is a hallmark of the Kondo effect that the correlated ground state explicitly involves the fermion reservoirs which are coupled to the nano-object. This opens the way to new physical effets when those reservoirs become for instance ferromagnetic, superconducting or themselves contain charge modes producing decoherence. Also, more complex nanostructures allows to explore intriguing related phenomena like quantum criticality, orbital Kondo effect or to investigate properties directly linked to the quantum coherence of the Kondo effect, like quantum interferences. Much work is expected on the experimental level to check and discover new behaviours, as well as to take advantage of some properties of the Kondo effect for spintronics devices.

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References

1 Kondo J. (1964) 'Resistance minimum in dilute magnetic alloys', Prog. Theor. Phys. Vol. 32, pp. 37-49.

2 Nozières P. (1974) 'A Fermi-Liquid description of the Kondo problem at low temperatures', J. Low Temp. Phys. Vol. 17, pp. 31-42.

3 Wilson, K.G. (1975), 'The renormalization group: Critical phenomena and the Kondo problem' Rev. Mod. Phys. Vol. 47, pp. 773-840.

4 Hewson A.C. (1997) 'The Kondo problem to Heavy fermions', Cambridge University Press, Cambridge (UK).

5 Ng T.K., Lee P.A. (1988) ‘On-site Coulomb repulsion and resonant tunneling’, Phys. Rev. Lett. Vol. 61, pp. 1768.

6 Glazman, L.I. , Raikh, M.E (1988) ‘Resonant Kondo transparency of a barrier with quasilocal impurity states’, JETP Lett. Vol. 47, pp. 452-455.

7 Glazman, L.I. , Pustilnik, M. (2004) ‘Kondo effect in quantum dots’, J. Phys. Condensed Matter Vol. 16, pp. R513-R537.

8 Goldhaber-Gordon D., Shtrikman H., Mahalu D., Abusch-Magder D., Meirav U., Kastner M.A. (1998) ‘Kondo effect in a single-electron transistor’, Nature (London) Vol. 391, pp. 156-159.

9 Cronenwett S. M., Oosterkamp T. H., Kouwenhoven L. P. (1998) ‘A Tunable Kondo Effect in Quantum Dots’, Science Vol. 281, pp. 540-544.

10 Rokhinson, L.P., Guo, L.J., Chou, S.Y., Tsui, D.C. (1999) ‘Kondo-like zero bias anomaly in electronic transport through an ultrasmall Si quantum dot’, Phys. Rev. B Vol. 60, pp. R16319-16321.

11 Specht M., Sanquer M., Deleonibus S., Guégan G. (2002) ‘Signature of Kondo effect in silicon quantum dots’, Eur. Phys. J. B Vol. 26, pp. 503-508.

12 Nygård J., Cobden D.H., Lindelof P.E. (2000) ‘Kondo physics in carbon nanotubes’, Nature (London) Vol. 408, pp. 342-346.

13 Park J., Pasupathy A.N., Goldsmith J.I., Chang C., Yaish Y., Petta J.R., Rinkoski M., Sethna J.P., Abruna H.D., McEuen P.L., Ralph D.C. (2002) 'Coulomb blockade and the Kondo effect in single-atom transistors', Nature (London) Vol. 417, pp. 722-725.

14 Liang W., Shores M.P., Bockrath M., Long J.R., Park H. (2002) 'Kondo resonance in a single-molecule transistor', Nature (London) Vol. 417, pp. 725-729.

15 Meir Y., Wingreen N. S., Lee P. A. (1993) ‘Low-Temperature Transport Through a Quantum Dot: The Anderson Model Out of Equilibrium’, Phys. Rev. Lett. Vol. 70, pp. 2601-2604.

16 Costi T.A. (2000) ‘Kondo effect in a magnetic field and the magnetoresistivity of Kondo alloys’, Phys. Rev. Lett. Vol. 85, pp. 1504; Costi T. A. (2001) ‘Magnetotransport through a strongly interacting quantum dot’, Phys. Rev. B Vol. 64, pp. 241310.

17 Moore J. E., Wen X.-G. (2000) ’Anomalous Magnetic Splitting of the Kondo Resonance’, Phys. Rev. Lett. Vol. 85, pp. 1722-1725.

18 Paaske J., Rosch A., Wölfle P. (2004) ‘Nonequilibrium transport through a quantum dot in a magnetic field: Perturbation theory’, Phys. Rev. B Vol. 69, pp. 155330.

19 Kogan A., Amasha S., Goldhaber-Gordon D., Granger G., Kastner M. A., Shtrikman H. (2004) ‘Measurements of Kondo and Spin Splitting in Single-Electron Transistors’, Phys. Rev. Lett. Vol. 93, pp. 166602.

20 Shiau S., Van Roermund R., Lavagna M. (2009) ‘Magnetic-field induced decoherence effects in a Kondo quantum dot’, unpublished.

21 Freyn A., Florens S. (2009) ‘Optimal broadening of finite energy spectra in the numerical renormalization group: application to dissipative dynamics in two-level systems’, Phys. Rev. B Vol. , pp. (R).

22 López R., Sánchez D. (2003) ‘Nonequilibrium spintronic transport through an artificial Kondo impurity: conductance, magnetoresistance, and shot noise’ Phys. Rev. Lett. Vol. 90, pp. 116602.

23 Martinek J., Utsumi Y., Imamura H., Barnás J., Maekawa S., König J., Schön G. (2003) ‘Kondo effect in quantum dots coupled to ferromagnetic leads’, Phys. Rev. Lett. Vol. 91, pp. 127203.

24 Pasupathy A.N., Bialczak R.C., Martinek J., Grose J.E., Donev L.A.K., McEuen P.L., Ralph D.C. (2004) ‘The Kondo effect in the presence of ferromagnetism’, Nature (London) Vol. 306, pp. 86-89.

25 Simon P., Cornaglia P.S., Feinberg D., Balseiro C.A. (2007) ‘Kondo effect with noncollinear leads: a numerical renormalization group analysis’, Phys. Rev. B Vol. 75, pp. 045310.

26 Ingold, G.L., Nazarov, Yu.V. (1992) ‘Charge tunneling rates in ultrasmall junctions’, in Single charge tunneling, ed. Grabert, H., Devoret, M.H., NATO ASI Series B, Vol. 294, pp. 21-107 (Plenum Press, New York).

27 Panyukov S.V., Zaikin A.D. (1991) ‘Coulomb blockade and nonperturbative ground-state properties of ultrasmall tunnel junctions’, Phys. Rev. Lett. Vol. 67, pp. 3168-3171.

28 Florens S., Simon P., Andergassen S., Feinberg D. (2007) ‘Interplay of electromagnetic noise and Kondo effect in quantum dots’, Phys. Rev. B Vol. 75, pp. 155321; Andergassen S., Simon P., Florens S., Feinberg D. (2008) 'Spin-orbital Kondo decoherence by environmental effects in capacitively coupled quantum dots', Phys. Rev. B Vol. 77, pp. 045309.

29 Altimiras C., Gennser U., Cavanna A., Mailly D., Pierre F. (2007) ‘Experimental test of the dynamical Coulomb blockade theory of short coherent conductors’, Phys. Rev. Lett. Vol. 99, pp. 256805.

30 Affleck I., Simon P. (2001) ‘Detecting the Kondo screening cloud around a quantum dot’, Phys. Rev. Lett. Vol. 86, pp. 2854; Simon P., Affleck I. (2001) `Persistent current around a quantum dot', Phys. Rev. B Vol. 64, pp. 085308.

31 Pereira R.G., Laflorencie N., Affleck I., Halperin B.I. (2008) ‘Kondo screening cloud and charge staircase in one-dimensional mesoscopic devices’ Phys. Rev. B Vol. 77, pp. 125327.

32 Simon P., Affleck I. (2002) `Finite size effects in Conductance Measurements on quantum dots', Phys. Rev. Lett. Vol. 89, pp. 206602; Simon P., Affleck I. `Kondo screening cloud effects in mesoscopic devices', Phys. Rev. B Vol. 68, pp. 115304.

33 Simon P., Salomez J., Feinberg D. (2006) ‘Transport spectroscopy of a Kondo quantum dot coupled to a finite size grain’, Phys. Rev. B Vol. 73, pp. 205325.

34 Schuster R., Buks E., Heiblum M., Mahalu D., Umansky V., Shtrikman H. (1997) ‘Phase measurement in a quantum dot via a double-slit interference experiment’, Nature (London) Vol. 385, pp. 417-420.

35 Ji Y., Heiblum M., Sprinzak D., Mahalu D., Shtrikman H. (2000) ‘Phase Evolution in a Kondo-Correlated System’, Science Vol. 290, pp. 779-783.

36 Ji Y., Heiblum M., Shtrikman H. (2002) ‘Transmission Phase of a Quantum Dot with Kondo Correlation near the Unitary Limit’, Phys. Rev. Lett. Vol. 88, pp. 076601.

37 Avinun-Kalish M., Heiblum M., Zarchin O., Mahalu D., Umansky V. (2005) ‘Crossover from mesoscopic to universal phase for electron transmission in quantum dots’, Nature (London) Vol. 436, pp. 529-533.

38 Zaffalon M., Bid A., Heiblum M., Mahalu D., Umansky V. (2008) ‘Transmission Phase of a Singly Occupied Quantum Dot in the Kondo Regime’, Phys. Rev. Lett. Vol. 100, pp. 226601.

39 Langreth D. C. (1966) ‘Friedel Sum Rule for Anderson’s Model of Localized Impurity States’, Phys. Rev. Vol. 150, pp. 516-518.

40 Gerland U., von Delft J., Costi T. A., Oreg Y. (2000) ‘Transmission Phase Shift of a Quantum Dot with Kondo Correlations’, Phys. Rev. Lett. Vol. 84, pp. 3710-3713.

41 Jerez A., Vitushinsky P., Lavagna M. (2005) ‘Theoretical Analysis of the Transmission Phase Shift of a Quantum Dot in the Presence of Kondo Correlations’, Phys. Rev. Lett. Vol. 95, pp. 127203.

42 Silvestrov P. G., Imry Y. (2001) ‘Spin effects and transport in quantum dots with overlapping resonances’, Phys. Rev. B Vol. 65, pp. 035309.

43 Golosov D. I., Gefen Y. (2006) ‘Transmission through quantum dots: Focus on phase lapses’, Phys. Rev. B Vol. 74, pp. 205316.

44 Karrasch C., Hecht T., Weichselbaum A., Oreg Y., von Delft J., Meden V. (2007) ‘Mesoscopic to Universal Crossover of the Transmission Phase of Multilevel Quantum Dots’, Phys. Rev. Lett. Vol. 98, pp. 186802.

45 Meden V., Marquardt F. (2006) ‘Correlation-Induced Resonances in Transport through Coupled Quantum Dots’, Phys. Rev. Lett. Vol. 96, pp. 146801.

46 Taniguchi T., Buttiker (1999) ‘Friedel phases and phases of transmission amplitudes in quantum scattering systems’, Phys. Rev. B Vol. 60, No 19, pp. 13814-13823.

47 Jarillo-Herrero P., van Dam J.A., Kouwenhoven L.P. (2006) ‘Quantum supercurrent transistors in carbon nanotubes’, Nature Vol. 439, pp. 953-956; van Dam J.A., Nazarov Yu.V., Bakkers E.P.A.M., De Franceschi S., Kouwenhoven L.P. (2006), ‘Supercurrent reversal in quantum dots’, Nature Vol. 442, pp. 667-670.

48 Jørgensen, H.I, Grove-Rasmussen K., Novotny, T., Flensberg K., Lindelof P.E. (2006), ‘Electron transport in single-wall carbon nanotube weak links in the Fabry-Perot regime’, Phys. Rev. Lett. Vol. 96, pp. 207003.

49 Cleuziou J.P., Wernsdorfer W., Bouchiat V., Ondarçuhu T., Monthioux M. (2006) 'Carbon nanotube superconducting quantum interference device', Nature Nanotechnology Vol. 1, pp. 53-59.

50 Winkelmann C. et al., unpublished.

51 Glazman L. I., Matveev K. A. (1989) 'Resonant Josephson current through Kondo impurities in a tunnel barrier', JETP Lett. Vol. 49, pp. 659-662.

52 Yoshioka T., Ohashi Y. (2000) 'Numerical Renormalization Group Studies on Single Impurity Anderson Model in Superconductivity: A Unified Treatment of Magnetic, Nonmagnetic Impurities, and Resonance Scattering', J. Phys. Soc. Jpn. Vol. 69, pp. 1812-1823.

53 Meng T., Florens S., Simon P. (2009) ‘Self-consistent description of Andreev bound states in Josephson quantum dot devices’, cond-mat/ 0902.1111; Meng T. et al., unpublished.

54 Nozières P., Blandin A. (1980) ‘Kondo effect in real metals’, J. Phys. France Vol. 40, pp. 193-211.

55 Jones B. A., Varma C. M., Wilkins J. W. (1988) ‘Low-temperature properties of the two-impurity Kondo Hamiltonian', Phys. Rev. Lett. Vol. 61, pp. 125-128.

56 Florens S., Rosch A. (2004) ‘Climbing the Entropy Barrier: Driving the Single- towards the Multichannel Kondo Effect by a Weak Coulomb Blockade of the Leads’, Phys. Rev. Lett. Vol. 92, pp. 216601.

57 Potok R. M., Rau I. G., Shtrikman H., Oreg Y., Goldhaber-Gordon D. (2007) ‘Observation of the two-channel Kondo effect’, Nature Vol. 446, pp. 167-171.

58 Zarand G., Chung C.-H., Simon P., Vojta M. (2006) ‘Quantum criticality in a double-quantum-dot system’, Phys. Rev. Lett. Vol. 97, pp. 166802.

59 Craig N. J., Taylor J. M., Lester E. A., Marcus C. M., Hanson M. P., Gossard A. C. (2004) ‘Tunable Nonlocal Spin Control in a Coupled-Quantum Dot System’, Science Vol. 304, pp. 565-567.

60 Hofstetter W., Schoeller H. (2002) ‘Quantum phase transition in a multilevel dot', Phys. Rev. Lett. Vol. 88, pp. 016803.

61 Roch N., Florens S., Bouchiat V., Wernsdorfer W., Balestro F. (2008) ‘Quantum phase transition in a single-molecule quantum dot’, Nature Vol. 453, pp. 633-637.

62 Withoff D., Fradkin E. (1990) ‘Phase transitions in gapless Fermi systems with magnetic impurities’, Phys. Rev. Lett. Vol. 64, pp. 1835-1838.

63 Fritz L., Florens S., Vojta M. (2006) ‘Universal crossovers and critical dynamics of quantum phase transitions: A renormalization group study of the pseudogap Kondo problem’, Phys. Rev. B Vol. 97, pp. 166802.

64 Dias da Silva L. G. G. V., Sandler N., Simon P., Ingersent K., Ulloa S. E. (2009) ‘Tunable pseudogap Kondo effect and quantum phase transitions in Aharonov-Bohm interferometers’, cond-mat/0901.2475.

65 Jarillo-Herrero P., Kong J., van der Zant H. S. J., Dekker C., Kouwenhoven L. P., De Franceschi S. (2005) ‘Orbital Kondo effect in carbon nanotubes’, Nature Vol. 434, pp. 484-488.

66 Le Hur K., Simon P. (2003) ‘Smearing of charge fluctuations in a grain by spin-flip assisted tunneling’, Phys. Rev. B Vol. 67, pp. 201308; Le Hur K., Simon P., Borda L., (2004) ‘Maximized Orbital and Spin Kondo effects in a single-electron transistor’, Phys. Rev. B Vol. 69, pp. 045326.

67 Borda L., Zarand G., Hofstetter W., Haleprin B. I., von Delft J. (2003) ‘SU(4) Fermi Liquid State and Spin Filtering in a Double Quantum Dot System', Phys. Rev. Lett. Vol. 90, pp. 026602.

68 Feinberg D., Simon P. (2004) ‘Splitting electronic spins with a Kondo double dot device’, Appl. Phys. Lett. Vol. 85, pp. 1846-1848.

69 Simon P., Feinberg D. (2006) ‘Electronic spin precession and interferometry from spin-orbital entanglement in a double quantum dot’, Phys. Rev. Lett. Vol. 97, pp. 247207.

70 Bohm D. (1951), ‘Quantum Theory’ (Prentice Hall, Englewood Cliffs, NJ).

71 Summhammer J., Badurek G., Rauch H. Kischko U., Zeilinger A. (1983), ‘Direct observation of fermion spin superposition by neutron interferometry’ Phys. Rev. A Vol. 27, pp. 2523-2532.